# Exercise

(The Geometric Model)

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## Contents



# 1 Exercises: The Geometric Model (VR Geometry)

Based on the concepts of 3D modeling, transformations, and orientations for Virtual Reality.

## 1.1 Level 1: Easy (Foundational Concepts)

1. [TH] Define a "Geometric Model" in the context of a Virtual World Generator (VWG).
   [SOL] A mathematical description of the shape, size, and position of objects in a 3D virtual space.
2. [NM] A point $P$ is located at $(2, 3, 5)$. If the model is translated by a vector $t = (1, -1, 2)$, what are the new coordinates of $P$?
   [SOL] $(2 + 1, 3 - 1, 5 + 2) = (3, 2, 7)$.
3. [SB] Explain why it is common to use triangles (meshes) to represent complex 3D objects instead of higher-order curved surfaces.
   [SOL] Triangles are computationally efficient, always planar, and supported by GPU hardware.
4. [TH] What are the three standard components of a 3D transformation (often abbreviated as TRS)?
   [SOL] Translation, Rotation, and Scale.
5. [NM] If a virtual world is defined in $\mathbb{R}^3$, what is the distance between point $A(1, 0, 0)$ and point $B(4, 4, 0)$?
   [SOL] $\sqrt{(4-1)^2 + (4-0)^2 + (0-0)^2} = 5$.
6. [TH] Define the difference between "Global Coordinates" and "Local (Body) Coordinates."

[SOL] Global: Fixed to world origin. Local: Fixed to the object; moves with it.

7. [SB] In a VR engine, you see an option to "Parent" a sword to a character's hand. Based on geometric modeling, what does this imply about their coordinate systems?
   [SOL] The sword's transform is relative to the hand's local coordinate system.
8. [NM] Express a translation of 5 units along the x-axis and -3 units along the z-axis as a 3D translation vector.
   [SOL] $t = (5, 0, -3)$.

## 1.2 Level 2: Medium (Transformations & Rotations)

9. [TH] Explain why 2D rotations can be represented by a single angle $\theta$, whereas 3D rotations require more complex representations (like matrices or quaternions).
   [SOL] 2D has only one axis; 3D rotations are non-commutative (order matters).
10. [NM] Write the $3 \times 3$ rotation matrix $R_z(\theta)$ for a rotation of 90° around the Z-axis.
    [SOL] $[[0, -1, 0], [1, 0, 0], [0, 0, 1]]$.
11. [SB] Discuss the phenomenon of "Gimbal Lock." Which rotation representation is most susceptible to it, and why is it a problem for VR head tracking?
    [SOL] Euler Angles. Occurs when two axes align, losing a degree of freedom.
12. [NM] Apply a $2 \times 2$ rotation matrix for $\theta = 45°$ to the 2D point $(1, 0)$. (Hint: $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$).
    [SOL] $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
13. [TH] What is a "Homogeneous Transformation Matrix," and what is the primary advantage of using a $4 \times 4$ matrix for 3D transformations?
    [SOL] A matrix that allows translation to be treated as a multiplication, simplifying composition.
14. [NM] Given a rotation matrix $R$, prove that its transpose $R^T$ is equal to its inverse $R^{-1}$ (the Orthogonality property).
    [SOL] $R \cdot R^T = I$ because rows/columns are orthonormal.
15. [SB] Compare the use of Euler Angles versus Quaternions for interpolating between two camera orientations. Why is one preferred for smooth movement?
    [SOL] Quaternions allow for SLERP (Spherical Linear Interpolation), which is smoother.
16. [TH] Define "Yaw," "Pitch," and "Roll" in the context of an aircraft or a VR headset.
    [SOL] Yaw: Y-axis; Pitch: X-axis; Roll: Z-axis.

## 1.3 Level 3: Hard (Advanced Modeling & Kinematics)

17. [NM] A camera is located at $(0, 0, 0)$ looking toward the negative Z-axis. An object is at $(0, 0, -10)$. If the camera moves to $(5, 0, 0)$ and rotates $90^\circ$ around the Y-axis (looking toward the positive X-axis), what are the object's coordinates in the *camera's new local frame*?
    [SOL] The object is now at local $(0, 0, 5)$ relative to the new camera orientation.
18. [TH] Describe the "Viewing Transformation." List the steps required to move from a 3D world coordinate to a 2D coordinate on the screen.
    [SOL] World $\rightarrow$ Eye $\rightarrow$ Canonical View $\rightarrow$ Screen.
19. [NM] Compose a single $4 \times 4$ homogeneous matrix that first rotates an object $30^\circ$ around the X-axis and then translates it by $(0, 10, 0)$.
    [SOL] $M = T \times R$ (Translation applied after rotation).
20. [SB] If a VR system has high latency in updating the "Geometric Model" relative to the user's head movement, describe the resulting visual artifacts and their impact on the user.
    [SOL] "Judder" or latency lag, leading to motion sickness.
21. [TH] Explain the concept of "Double Covering" in the context of Unit Quaternions ($q$ and $-q$ representing the same rotation).
    [SOL] Two points on the 4D hypersphere represent one 3D rotation.
22. [NM] Convert the quaternion $q = (\cos(\theta/2), 0, 0, \sin(\theta/2))$ back into its equivalent rotation matrix form.
    [SOL] Result is the standard $R_z(\theta)$ matrix.
23. [SB] You are designing a VR "Mirror" where the user sees their avatar. Explain the geometric transformation required to reflect the avatar's movements across a plane $x = 0$.
    [SOL] Reflection matrix where $x' = -x$.
24. [NM] Calculate the result of rotating point $v = (1, 0, 0)$ by $180^\circ$ around the Y-axis using the quaternion formula $v' = qvq^{-1}$.
    [SOL] $v' = (-1, 0, 0)$.
25. [TH] In the "Rendering Pipeline," explain the role of the "Culling" process.
    [SOL] Removing objects outside the Viewing Frustum to optimize performance.